Inside: Multiple Intelligences ▶ Logical-Mathematical Intelligence ▶ Logical and Mathematical Intelligence in Montessori ▶ Why buy plastic multi-coloured gadgets for infants? ▶ External order for toddlers ▶ Concrete math in the Children’s House ▶ A lot of thinking in Primary! ▶ The Secondary Section ▶ The logical - mathematical intelligence alive at home

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Howard Gardner is the developmental psychologist who put forward the idea of the eight multiple intelligences. All of the intelligences are equally important. A well-rounded functioning person has had the opportunity to develop all of them. Even though standard schools believe that their effectiveness can be measured by tests such as the PISA test, they forget that a successful person in life and work needs a lot more than facts, concepts and memorization. By incorporating a mixture of activities into the learning experience, that meet the eight different styles of learning, one ensures maximum effectiveness, input and retention. One also assists in the development of skills such as, communication, empathy, understanding, responsibility, and conflict resolution. These only develop when the characteristics of the different intelligences have been integrated. This integration causes ‘cross-fertilization’ which is the secret of a successful human being.
Logical-mathematical intelligence consists of many factors related to the analytical, synthetic and integration functioning of the mind. When developed well the person becomes a divergent thinker. Traditionally, we think math provides these capacities, but that is of course not the only subject that helps logical thinking. In principle, when children are empowered and interested, every topic can help the capacity to: structure, analyse logically, investigate issues, recognize patterns, question critically, reason deductively and come to conclusions by integrating information. In Howard Gardner’s words, the logical-mathematical intelligence entails the ability to detect patterns, reason deductively and think logically.

Mathematics is one of the subjects that assist in the development of logical-mathematical intelligence. However, to develop logical thinking, it should not be taught to young children in its traditional form, i.e. starting at the beginning of a work book and going through this book from cover to cover.
during the course of the school year. Math skills and concepts need to be presented in a concrete form so that these can be experienced firsthand and then integrated with other knowledge. The patterns learned become part of a logical thinking process. When this is done in small groups, sometimes with children who are on the same level and sometimes with those of different levels, they learn from each other, communicate, are given math problems, find solutions, and learn to help and explain to others. That way, children explore and cultivate a diversity of skills and knowledge that match what they need in order to function successfully.

**Strengths particular to this intelligence**

- Enjoys the process of figuring things out
- Is orderly
- Asks questions
- Works with numbers, measurements, degrees, angles, dimensions etc.
- Likes conducting scientific experiments in a logical manner
- Explores patterns and relationships
- Good problem-solving skills
- Enjoys thinking about abstract ideas
- Good at solving complex situations
- Organises him/herself by classifying and categorizing information,
- Questions and wonders about natural events
- Pursues ideas
- Fascinated with patterns within different subject knowledge
- Interested in the how: How does it function, how is it possible, how do we go about it?
- Has good ability for abstract thought
- Good intuition, integrates and draws from several areas of the brain.

**Possible Career choices**

- Many different professions that require an organised, orderly person with logical and abstract thinking skills
- Specific to the subject mathematics:
  - Scientists
  - Mathematician
  - Computer programmer
  - Engineer
  - Accountant
  - Architect
  - Research and Development
  - Law
Logical and Mathematical Intelligence in Montessori

Howard Gardner is the ‘master-mind’ behind the terminology and elaboration of the theory of the 8 intelligences. He aimed for describing the learning styles and diversity in functioning of human beings, not only to help people in specific, but also to help the world ahead and create the conditions to change it. Because, when education does not change, then thinking patterns might not change either. So how will the world change?

Maria Montessori understood these concepts very well. She developed an educational tool for teachers in order to bring this theory in practice. The Montessori classroom is a mini-society in which the child can develop all characteristics.

Children can learn to think logically due to many factors:
- An organised classroom that offers multiple experiences
- Having the opportunity to develop true understanding by using knowledge gained in one setting and use it in another
- Moving from concrete to abstract understanding
- Emphasis on all aspects of the development since one aspect influences the other.
- Being empowered with the possibility of choice
- Take initiative and be creative
- Working in different social structures and with students who are at different levels; diversity creates flexibility.
House, an integrated knowledge at the Primary level, technology in the MYP, and extended essays at the DP level all contribute to the development of logical-mathematical intelligence. Howard Gardner has a controversial saying in that a person can actually be bad at math and have a very well developed logical-mathematical intelligence!

- Developing empathy due to multiple and diverse interactions
- Exposed to an integrated curriculum, offering 360-degree view of the world and society at large.

Now let’s have a look at the different age ranges. Knowing that logical thinking derives from many experiences and factors as described above, this newsletter will focus on the contribution of the classroom activities and interactions as a whole and in specific of the math curriculum to the development of the logical-mathematical mind. But whilst reading about mathematics, please do not forget that an ordered environment at the infant level, practical life experiences in the Children’s
The base for logical-mathematical intelligence is order. When a baby is born, the brain is busily developing its neuro-networks in response to nature (genes) and nurture. Nurture consists of the physical environment, which is at that stage the home environment, and the psychological environment. These are the parents, siblings and other caretakers; their language, but also their emotional life, values, cultural preferences, judgments and ideas are all absorbed by the infant. Thus becoming a member of his/her family and culture.

As the child absorbs information, it needs to be ordered into categories. A categorized brain is an efficient and affective brain. When information is offered in a muddled-up version, e.g. when the TV is always on and gives background noises to language interactions, the brain will need more energy in classifying that new knowledge.

When toys are offered in an organised and limited manner, the brain stays calmer, the child can intentionally move from one to another object, explore it, store related information, change focus and repeat the action. Compare this to a room full of multicoloured plastic objects that beep, flash and make crunchy noises. The brain receives a multitude of non-related and non-logical information at the same time. Again, analysing and categorising becomes more difficult.

A calm, ‘measured’ environment that is orderly and moves along with the natural pace of the infant offers the best possible start to the development of all intelligences. The logical-mathematical mind can start developing immediately as it does not need to use unnecessary energy to block out other non-related input. Whilst the child is slithering to that one important object, gross motor develops, the mind learns to focus, the will develops, independence is increasing, the object is explored and all information gets sorted in the neuro-network.

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Why buy plastic multi-coloured gadgets for infants?
External order remains important up to the age of six. The child uses order in the environment to develop internal mental order. Whilst the child is building up experiences through exploration, interaction and language communication, the brain forms its networks. It also prunes away the neuro-pathways that are not needed anymore. Order (or disorder) gets absorbed and becomes part of the neuro-network. A brain can become ‘overly-wired’ and sometimes one talks about ‘wired-up’ of ‘over-active’ people. The two concepts can be very much related. The brain can link incredibly fast and can become very creative and inventive. However, the better experiences and knowledge can be classified and stored away, the more efficient the brain becomes.

An environment that has an order and functions logically gets absorbed as such. This order is related to objects and the functioning of the other people within it. Logical thinking can also be stimulated with planned activities that match the interest of the child. Initially that revolves around the basic concepts such as looking after oneself, the environment and others. So dressing oneself, whilst choosing from a child size cupboard that offers a limited selection of clothes, highly influences logical thinking. Cooking activities that involve several steps that are organised according to the capacity of the child, develops step-by-step thinking. Singing, counting and reading also helps as these activities influence the concept of sequence.

Adults analysing activities, and ‘climbing into the children’s brain’ before offering it to them greatly helps the child to learn to analyse and synthesise. Many activities bought in shops are too involved. Sometimes, by taking half of it out, the activity gives much more pleasure. The learning opportunity then happens when offering the whole box full.

Toddlers do not need TV or computers. They need ‘child-size’ activities that they can do with their hands. The hands are the tools to the formation of the brain. When they work they build up real-life experiences that can be classified and stored. Thus helping them in becoming logical, reality based people.
Jean Piaget developed his theory on educational experiences at the same time as Maria Montessori. His main topic was researching the logical-mathematical intelligence. He came to the conclusion that this logical-mathematical intelligence derives from the handling of objects, grows into the ability to think concretely about those objects, terminology is added, and then develops into the ability to think formally of relations without objects.

Based on this line of thinking, a very concrete start in providing math concepts is available to children in the Children’s House. By the end of this age-range, they actually handle numbers up to 9999, which is much further afield than is traditionally the case. For them, 9999 is not an abstract number, it is represented by 9 thousand cubes, 9 hundred squares, 9 ten bars and 9 unit beads. This material is called the golden beads. They have been given the colour gold to represent its importance; the introduction and functioning of the decimal system.

With this material children perform additions, subtractions, multiplications and divisions. They perform the actions and also start with the memorisation of the basics of these operations.

Much earlier, at the age of three, children develop eye-hand coordination in relation to counting with the number rods. These rods are subdivided by
pieces of 10 cm that are alternatively coloured red and blue. One piece of 10 cm long represents the number one. Ten attached represent the number ten and are 1 meter long. This sounds tedious, but look at the hidden information given. The child learns to count up to 10, but already ‘knows’ what one meter is. This is of course pointed out later, in Primary, but this is a nice example of the multifaceted learning within the Montessori material:

- The child goes through a concrete experience
- He/she develops eye/hand coordination
- The related language is presented and the child learns to count to ten
- The child stores in the long-term memory the ‘feel’ of 10 cm, 20 cm and so on to one meter.
- This indirect preparation is brought to the surface at a later stage and the child has the ‘feeling’ he/she knew this all along.
- Based upon the concrete exploration, abstract concepts are formed
- The child feels secure with new knowledge.

In between the two above described levels, the child works within several consecutive levels and different related materials. The sequence of these presentations has been very well thought through. Each time, the child learns something new, based on something he/she already knows. Logical and mathematical intelligence develops accordingly, since there is no stress, pressure and insecurity. Logical thinking develops along with every step containing new exploration possibilities and additional isolated informational facts that can then become integrated.

The Children’s House also offers sensorial activities that help the development of the senses. People, who use all their senses and see what there really is to see, come to different conclusions. Observation is an important tool of the logical-mathematical intelligence, and therefore it is given its due attention in the environment.

Practical Life activities offer practice and a step-by-step way of thinking. The child needs to analyse how to clean the painting easel, then perform and come to the conclusion that if you do not rinse your sponge in clean water, you will actually never get the easel clean enough. Cause and effect activities help the development of logical thought. Experience makes the difference; being told is not concrete enough. Each child needs to go through the practical experiences, to be able to build the mental framework.

Language is presented first at an oral level and then later on a written level. Children increase vocabulary through the language activities itself and by means of projects related to early geography, biology, botany and some history. Songs, books and stories are added. Precision in language influences logical thinking. This also works the other way around; logical thinking influences precision in language. This is why attention to language development is much more important than only for the language itself.
The Primary children are ready to expand their thought structure. At this stage the absorbent mind has done it’s work and effortlessly absorbed the world around them. That specific functioning of the mind slowly decreases and changes to a reasoning mind. A person can only reason when he/she has enough reasoning material. Reason is choice; it is a choice of different possibilities. This is also why choice is given within the classroom structure. When you can choose, you have to think about the pros and cons. So the mind jumps into action. Choice becomes decision and therefore children are given decision power in the classroom.

This level of ‘power’ depends on age and ability. Decision making needs to be practised in order to become efficient.

The multi-age range offers work being done at different levels. This is a wonderful aspect for the growing logical-mathematical mind. The children have other thinkers as role models around them. Since those are also children, it is very accessible and non-threatening. This stimulates children to move to a higher, more advanced thinking level.

And then there is the integrated curriculum. There is no better tool for the development of this intelligence. Students learn to think about a topic from many points of view. When subjects are separated, students practice linear thinking. However, when these are presented holistically, students can move toward divergent thinking.

Learning to see items from 360 degrees influences again decision making and solution finding. Those are two very important consequences of the logical-mathematical mind.
Now let’s look at the math curriculum itself, since it is beautifully designed to influence this intelligence. Mathematics has several components that run parallel to each other and influence each other’s level. These are:
Hierarchies
Memorisation
Algebra
Geometry
New math

The information, knowledge and skills given to the children can be visualized as the shape of a funnel. As the base gets wider, the children learn more and integrate the different components.

The students start with hierarchies. They learn to do the four operations with materials which are more abstract than as discussed in the Children’s House section. First they learned to do these operations on a static level (no carrying or borrowing), now it becomes dynamic. Always trading the higher hierarchy in order to perform the action. At the same time memorisation has started. There are many different math finger-charts and boards that help memorisation. Also math games and real-life situations (shopping and use of money at home) can really help. As the child is progressing, larger numbers are presented. Long divisions and cross multiplications start to happen. They also move to non-whole numbers (fractions, decimals and percentages).

Around the age of seven or eight algebra is slowly introduced, first at a concrete and understandable numerical level. They make decanomials, go from one square to the following, square root and eventually the cube root. With this they learn about relationships of numbers. At the end of Primary and beginning of Secondary this moves into balancing simple equations using the laws of equivalence.

Geometry happens simultaneously. Since the children learn within a concrete integrated curriculum they start to understand the relationships. After having learned names of shapes in Children’s House and concretely experienced triangles according to sides and angles, they now move to the conscious level and children learn all about lines, angles, plane figures, triangles, quadrilaterals, polygons and circles. Important work is done in relation to congruency, similarity and equivalence, which is later needed for the calculation of area. At the end of Primary and beginning of Secondary volume is being discussed.

New Math offers calculations in different bases. This is a topic that helps integrate the knowledge of the decimal system. Once a students understands that thoroughly he/she can calculate in any base, or maybe calculate the Egyptian way when they study the Pharaohs!

Arithmetic is offered in a very concrete manner and slowly becomes more and more abstract. The child then weans off of the materials and can work purely in the abstract. Then textbooks, calculators, compasses, protectors and computers can take over. That is what happens in the Middle Years Programme.
The Middle Years Programme and the senior secondary students are exposed to a wide curriculum offering several languages, humanities, sciences, the arts, sports and of course math. These older students continue to develop their logical thinking skills and slowly incorporate more and more abstract knowledge. However, especially at this age, communication, mentorship, interrelationships, debating, reflective moments, interaction with older and younger students is extremely important for the development of the whole personality.

Now they move towards the next stage of development. They want to know how the society works, how relationships work within the society and what will be their role and task in the future society. This they cannot learn only behind desks and in large groups of people who are of the same age and thinking level. True personal and small group interactions are necessary so that they can come to healthy logical decisions. Here one can truly see the logical-mathematical mind at work. The consequence of an adolescent who is not functioning on a logical level becomes more apparent than with a small child. It is not about being willful anymore and not eating their soup; instead it can show itself in an eating disorder. The small child may have difficulty recognising the needs of others while this manifests itself in an adolescent as having difficulty maintaining healthy relationships.

So the following ingredients help teenagers to develop logical-mathematical thinking:

• Small group interactions
• Caring adults around them who help with their whole development
• Personal responsibility
• Mentorship
• Limits according to their age and capacity
• Learning from logical consequences
• Interrelated curriculum through guiding questions
• Continuous development of abstract thought
• Continuous development of skills needed to lead an effective and happy life
• Additional knowledge and understanding on how society functions.

Mathematics is given within this structure and offers the continuation of the development of abstract thought within the world of numbers.
Math in the Middle Years Programme

The curriculum in the MYP is divided into five branches being:

- Numbers
- Algebra
- Geometry and Trigonometry
- Statistics and probability
- Discrete mathematics

As in the Primary, students in MYP Years 1 and 2 continue learning concepts through sensorial materials that develop into real world applications. A higher passage to abstraction is achieved from the levels of abstraction already reached in the second plane of development. Thus, a synthesis is reached between the hand and head so that the student can solve problems using a reasoned approach.

Mathematics lessons are held for one hour three times per week. The first 20 minutes involve learning a concept. The students then perform guided autonomous work during the remaining time. Work that is not completed at school is continued at home. Guidance and input of the parents is needed and offers an added opportunity to the children.

A cycle of lessons includes three phases. The first phase is the introduction of a concept, it provides new vocabulary, or initiates a challenge. This is followed by the second phase in which the students engage in exploration, discovery, inquiry, research, activities and/or practical tasks. During the final phase students demonstrate mastery through knowledge and understanding, testing, communication and/or reflection.

Students have the opportunity to develop logical, critical and creative thinking, patience and persistence in problem solving. They then can apply and transfer these skills to a wide range of situations including real life, other areas of knowledge and future developments.

To ensure a smooth transition of Primary students to the MYP, students are guided through a review of mathematics, mentored in time management and self-management, note taking, collaboration, and effective communication. These life skills will bring valorisation, competence, self-confidence, and trust within a nurturing environment.

Contribution by Brian Thelen

Brian teaches math to the students of Year 1 and 2 of the Secondary Section.

Relating the subject of math to the guiding questions

A teacher once claimed that forgetting things meant you didn’t find the request important enough to remember. Important things will stick to you and don’t require effort to recall it.

Growing up I learnt that any subject could come to life when finding points of contact between subject matter and the learner. We just need to be touched or linked personally in some way.

Imagination, visualisation, emotions and senses can bring you far. You might still forget the little things, but boredom and indifference will be replaced by the eagerness to learn. Effort will not make you tired anymore, but will give fulfilling knowledge and internal growth.
Additionally to the work in relation to the guiding question, students work on the continuous math curriculum. Everybody should have a good mathematical understanding in order to be able to move forward to an independent problem solving level and to have the confidence and self-esteem to get a step further. All students can work on this at his/her own pace during autonomous work time. Children are mentored during this time and progress is being monitored. The main emphasis is on understanding and logical thinking. After getting through the basic skills everyone will be pushed as far as they can get. So there will be plenty of differentiation on our way!

**Contribution by Leila Ibrahim**

*Leila teaches math to the students of Year 3 to 6 of the Secondary Section.*

Therefore our current interdisciplinary unit of ‘Resources and Recycling’ is a very rewarding subject, since it allows me, as math teacher, to enter various mathematical skills in a large field of environmental topics such as awareness of individual environmental contributions and problem solving calculations for a better environment.

In order to stimulate the students further, my starting point was a discussion on water scarcity. Students have to think about the following issues: Isn’t there enough water on earth? Can we use salt water as drinking water? Do you know how to make salt water drinkable? Where can we use this procedure?

The discussion will be followed by 3 problem-solving exercises. In order to solve these, students first need to incorporate different mathematical skills such as:

- Working with powers
- Standard form
- Significant numbers
- Conversion between standard sized cubes and the main metric unity of capacity
- Calculation of volumes
- Working with fractions, percentages and decimals

After the first introduction discussion on the vulnerability of water, we brainstorm on our own water use and how to reduce it. Students will have to collect relevant and useful information on the Internet. They need to demonstrate their mathematical skills:

- Percentage increase and decrease
- Statistics: pie charts, for the older one’s cumulative frequency polygons and histograms in their extended independent work
- Upper and lower bound of rounded numbers (not for Year 3)
- And other problem solving exercises

The students will produce a piece of independent work about the financial profits when using a water tank that catches the rain water. The older students will produce an extended piece of independent work, about solar panels, as well.
The logical mind develops with everything the child does in an organised structured manner. Activities need to be at the level that is appropriate to the age and related capacity of the brain. So the rule is—not too much and not too little. What a young child, up to the age of 6 can see and handle in real life, is what he/she can handle theoretically. So one can talk about flowers, animals, insects, islands, lakes, zoo, products in shops, stars, the moon, transport, etc. One cannot discuss pollination, moon rotation, seasons due to earth rotation, the functioning of a cylinder and so on. Wait, these abstract subjects are for six years and older.

It is easy for us to space our own language and gauge the information given to children when we speak to them. However, the TV does not do that. Much of television programming and television formatting is not conducive to the child’s brain development. Even when the content seems harmless, the brain cannot sort and filter quick edits, sound effects, and background music. The information is cluttered and not comprehensible in real life terms. It ‘clogs’ the system. Consequently quiet children become quieter and active children become overactive. The same happens when the adults talk too much, talk too little or give too much detailed information.
The child then does not know how to categorise the information. It is too foreign. One can see this in children who watch too much and inappropriate programmes on TV, console games or computer games. Repeatedly they act out in the playground what they see. By doing this they try to give it a concrete base, since that is storable. However, the whole process interferes with the healthy process of absorption.

Real life activities and personal interaction is fundamental. Language is an important tool in order to help the development of the logical-mathematical mind.

The following aspects assist in this development:

- For children up to the age of approximately six, do not give more information than what is concretely visible in their world. The mind absorbs everything around him/her and this information needs to be classified first. The mind is not ready yet to imagine on top of this, since the acquisition of basic knowledge of the world is still in process. Around the age of six an active and attentive child has accumulated enough information to start to develop imagination. From then, adults can give new knowledge that is not immediately tangible.
- Listening when children express their thought
- Also stopping them when they talk continuously/too much
- Insist on the ‘dance’ of language; talk, listen respond, then the other person talks, listens responds. This balance of equality between the two communicators helps the child to frame his/her thought in a ‘language acceptable’ time frame.
- Insist on a verbal response when you ask something. Especially children who tend to speak less, do not let them get away with a ‘grunt’
- Always make and insist on eye contact when communicating
- When communicating, try not to have other interruptions at the same time i.e. no radio, TV, GSM ringing, computer activity
If they do not do their homework in MYP, parents do not need to excuse them. They again need to think and find away to go about this. Learning from logical consequences is usually very efficient. These experiences will help the development of logical thinking, which in its turn influences the development of responsibility. Since the latter will not develop without the first.

Exercises to strengthen logical-mathematical intelligence:

For younger children:

- Offer an organised environment
- Offer a logical organisation of belongings such as clothing, books and toys
- Have a calm surrounding
- Limit exposure to TV, as this offers very limited life experiences.
- Turn off the TV/radio when the child does other activities
- Offer activities with which the child needs to use and develop all senses
- Concrete materials are the link to the development of the abstract ideas
- Help children to talk clearly, enunciated speech. Precision in language enhances precision in thought.
- Say ‘no’ when you feel uncomfortable
Ask children to use a tool/equipment/furniture only for what it is meant to be.
Give choice out of two options when appropriate: shoes or sandals, mittens or gloves.
Do not let children make ‘life decisions’, they do not have enough life experience.
Provide for lots and lots of appropriate concrete activities. The brain develops through the hands.

Good Google list:
- www.infed.org/thinkers/gardner.htm
- www.inspiring-breakthrough.co.uk/learning-styles/mathematical-logical-learning.htm

Reading list:
- The Disciplined Mind by Howard Gardner
- Math for Humans by Mark Wahl
- Piaget for Educators, by R. Bybee and B. Sunday
- Inside the Brain, by Ronald Kotulak

For older children:
- Practise step-by-step thinking
- Insist on organisation of belongings, files, homework
- Have a place for everything
- Say no when you feel uncomfortable and insist.
- Let logical consequences do their job
- Predict before you calculate
- Select key points in a text
- Read carefully and reason
- Brainteasers provide great practise
- Do practical experiments within other subjects,
- Collect, analyse and interpret data
- Play games such as Cluedo, Mastermind, monopoly, chess
- Visit a science museum
- Talk through logical or mathematical problems
- Unravel the logic of a mystery story
- Open the broken down printer and discover how it works, find the pieces, reassemble
- Write stories, in there you think, plan, organise, and perform revisions.
- They can handle bigger decisions now, but not ‘life savers’; big decisions and related limits need to be set by the parents. Teenagers still need a consistent framework in which they feel secure and supported. With logical limits appropriate to their age and without the stress and responsibility of big decisions, the logical mind can continue to develop harmoniously.